Class X Session 2025-26
Subject - Mathematics (Basic)
Sample Question Paper - 05

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

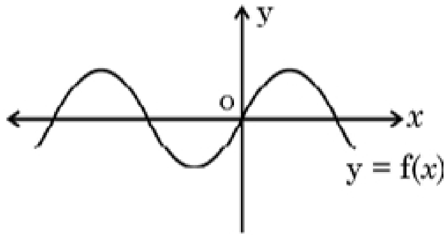
Section A

1. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number is: **[1]**

a) 36 b) 45

c) 81 d) 9

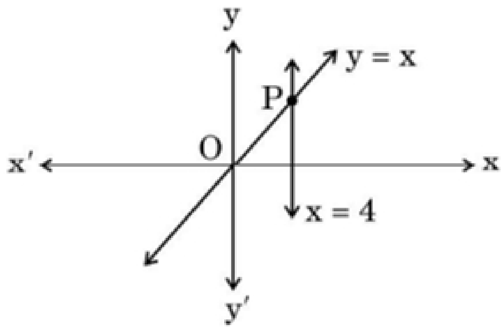
2. The graph of y = f(x) is shown in the figure for some polynomial f(x). The number of zeroes of f(x) are **[1]**



a) 2 b) 1

c) 3 d) 4

3. The lines represented by the linear equations y = x and x = 4 intersect at P. The coordinates of the point P are: **[1]**



a) (4, 4)

b) (-4, 4)

c) (4, 0)

d) (0, 4)

4. $4x^2 - 20x + 25 = 0$ have **[1]**

a) No Real roots

b) Real roots

c) Real and Equal roots

d) Real and Distinct roots

5. The sum of the first 21 terms of an A.P 16, 12, 8, 4, ... is: **[1]**

a) -484

b) -504

c) -480

d) 1176

6. The distance between the points A(-1, 5) and B(6, -2) is: **[1]**

a) 49

b) $7\sqrt{2}$

c) $2\sqrt{7}$

d) 14

7. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then **[1]**

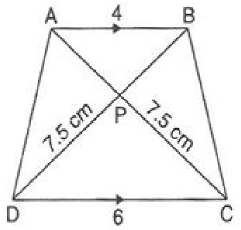
a) $AP = \frac{1}{3}AB$

b) $AP = \frac{1}{4}AB$

c) AP = PB

d) $AP = \frac{1}{2}AB$

8. In the given figure, if AB||DC, then AP is equal to **[1]**



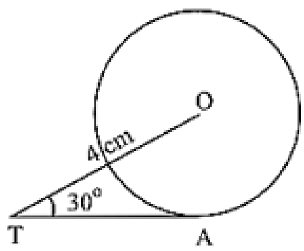
a) 7 cm.

b) 5 cm.

c) 5.5 cm.

d) 6 cm.

9. In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, ∠OTA = 30°, then length of TA is: **[1]**



a) $\sqrt{3}$ cm

b) $2\sqrt{2}$ cm

c) $2\sqrt{3}\ cm$ d) 2 cm

10. To draw a pair of tangents to a circle, which are inclined to each other at an angle of 45°, we have to draw tangents at the endpoints of those two radii, the angle between which is **[1]**

a) 145° b) 140°

c) 105° d) 135°

11. The value of 2 cos 45^o cot 30^o is **[1]**

a) $\sqrt{6}$ b) $2\sqrt{3}$

c) $\frac{\sqrt{6}}{2}$ d) $\frac{\sqrt{3}}{2\sqrt{2}}$

12. If $\sin\theta = \frac{3}{4}$, then $\frac{(\sec^2\theta-1)\cos^2\theta}{\sin\theta}$ equals: **[1]**

a) $\frac{9}{16}$ b) $\frac{3}{4}$

c) $\frac{4}{3}$ d) $\frac{3}{5}$

13. The ratio between the height and the length of the shadow of a pole is $1{:}\sqrt{3}$, then the sun's altitude is **[1]**

a) 60° b) 75°

c) 45° d) 30°

14. Area of a sector of angle θ (in degrees) of a circle with radius r is: **[1]**

a) $\frac{\theta}{180} \times \pi r^2$ b) $\frac{\theta}{180} \times 2\pi r$

c) $\frac{\theta}{360} \times 2\pi r$ d) $\frac{\theta}{720} \times 2\pi r^2$

15. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π cm^2, then find it's radius **[1]**

a) 16 cm b) 10 cm

c) 12 cm d) 8 cm

16. If probability of winning a game is p, then probability of losing the game is: **[1]**

a) p - 1 b) 1 + p

c) -p d) 1 - p

17. Two dice are thrown simultaneously. The probability of getting a doublet is **[1]**

a) $\frac{2}{3}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{6}$

18. Mean and median of some data are 32 and 30 respectively. Using empirical relation, mode of the data is: **[1]**

a) 30 b) 26

c) 36 d) 20

19. **Assertion (A):** A piece of cloth is required to completely cover a solid object. The solid object is composed of a hemisphere and a cone surmounted on it. If the common radius is 7 m and height of the cone is 1 m, 463.39 cm^2 is the area of cloth required. **[1]**

Reason (R): Curved Surface area of hemisphere = $2\pi r^2$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** Common difference of an AP in which $a_{21} - a_7 = 84$ is 14 **[1]**

Reason (R): nth term of AP is given by $a_n = a + (n - 1)d$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

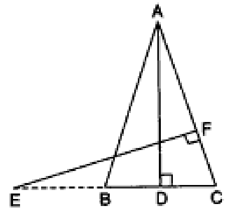
c) A is true but R is false.

d) A is false but R is true.

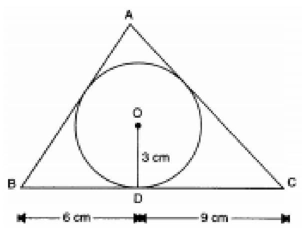
Section B

21. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number. **[2]**

22. In the figure, E is the point on side CB produced on an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF$\perp$ AC, prove that $\triangle ABD \sim \triangle ECF$. **[2]**



23. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle ABC$ is 54 square centimeter, then find the lengths of sides AB and AC. **[2]**

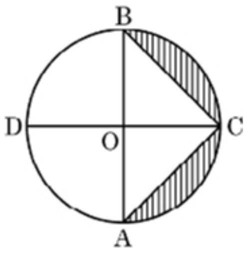


24. Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$ = cosec A + cot A **[2]**

OR

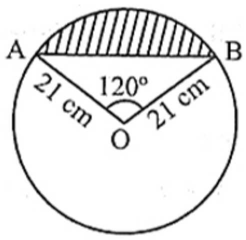
Find the value of θ, if, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \leq 90^\circ$.

25. In the given figure, AB and CD are the diameters of a circle with centre O, perpendicular to each other. If OA = 7 cm, find the area of the shaded region. **[2]**



OR

Find the area of the segment shown in Fig., if radius of the circle is 21 cm and $\angle AOB = 120^o$ (Use $\pi = \frac{22}{7}$)



Section C

26. Show that $3 + 5\sqrt{2}$ is an irrational number. **[3]**

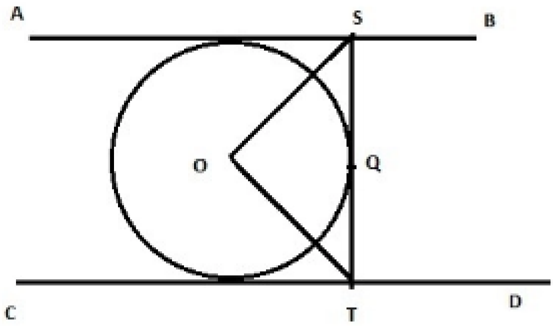
27. If α, β are zeroes of the quadratic polynomial $x^2 + 9x + 20$, form a quadratic polynomial whose zeroes are $(\alpha + 1)$ and $(\beta + 1)$. **[3]**

28. In an AP, the first term is 22, nth term is -11 and sum of first n terms is 66. Find n hence find the common difference d. **[3]**

OR

For what value of n are the nth terms of the following two APs the same 13,19,25, ...and 69,68,67,...? Also, find this term.

29. In the adjoining figure, AB and CD are two parallel tangents to a circle with centre O. ST is the tangent segment between two parallel tangents touching the circle at Q. Show that $\angle SOT = 90^\circ$ **[3]**



OR

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

30. Prove that (cosec A - sin A) (sec A - cos A) = $\frac{1}{\cot A + \tan A}$. **[3]**

31. Find the mean, median and mode of the following data: **[3]**

Classes:	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Frequency:	2	3	5	6	5	3	1

Section D

32. If the equation $\left(1 + m^2\right) x^2 + 2mcx + \left(c^2 - a^2\right) = 0$ has equal roots, prove that $c^2 = a^2 \left(1 + m^2\right)$ **[5]**

OR

If the factory kept increasing its output by the same percentage every year. Find the percentage, if it is known that the output doubles in the last two years.

33. If the angle of elevation of a cloud from a point 10 meters above a lake is 30^o and the angle of depression of its reflection in the lake is 60^o, find the height of the cloud from the surface of lake. **[5]**

34. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹10 per dm^2. **[5]**

OR

A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy. (Use $\pi = \frac{22}{7}$ and $\sqrt{149}$ = 12.2)

35. The table shows the daily expenditure on food of 25 households in a locality: **[5]**

Daily Expenditure (₹)	100-150	150-200	200-250	250-300	300-350
Number of Households	4	5	12	2	2

Find the mean daily expenditure on food. Also, find the modal expenditure.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Two schools **P** and **Q** decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School **P** decided to award a total of ₹ 9,500 for the two games to 5 and 4 students respectively; while school **Q** decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.

i. Represent the following information algebraically (in terms of x and y). (1)
ii. What is the prize amount for hockey? (1)
iii. Prize amount on which game is more and by how much? (2)

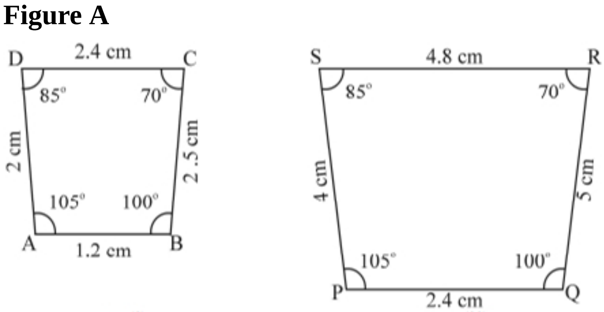
OR

What will be the total prize amount if there are 2 students each from two games? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

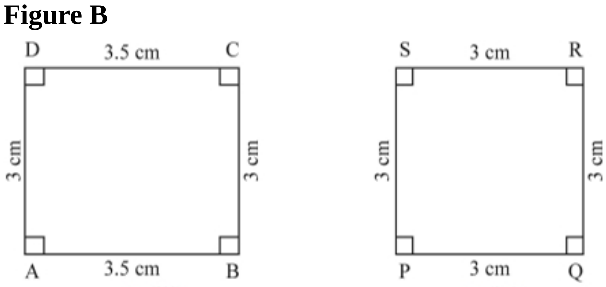
Observe the figures given below carefully and answer the questions:

Figure A



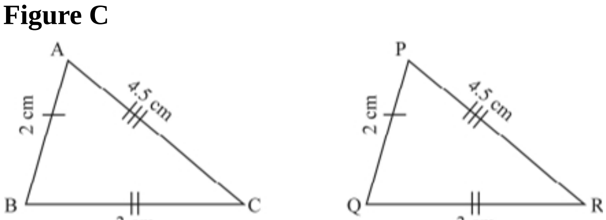
A (i) A (ii)

Figure B



B (iii) B (iv)

Figure C



C (v) C (vi)

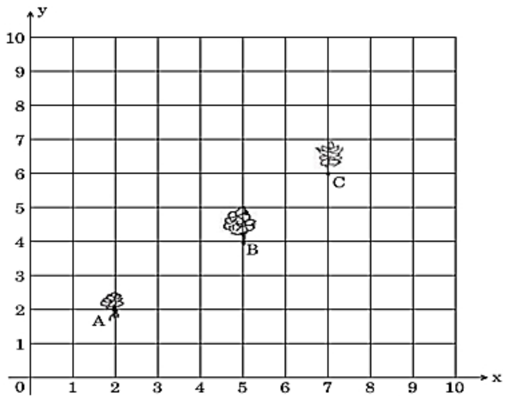
i. Name the figure(s) where in two figures are similar. (1)
ii. Name the figure(s) wherein the figures are congruent. (1)
iii. Prove that congruent triangles are also similar but not the converse. (2)

OR

What more is least needed for two similar triangles to be congruent? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Reena has a 10 m $\times$ 10 m kitchen garden attached to her kitchen. She divides it into a 10 $\times$ 10 grid and wants to grow some vegetables and herbs used in the kitchen. She puts some soil and manure in that and sow a green chilly plant at A, a coriander plant at B and a tomato plant at C. Her friend Kavita visited the garden and praised the plants grown there. She pointed out that they seem to be in a straight line. See the below diagram carefully:



i. Find the distance between A and B? (1)

ii. Find the mid-point of the distance AB? (1)

iii. Find the distance between B and C? (2)

OR

Find the mid point of BC. (2)

Solution

Section A

1.

(c) 81

Explanation:

Let the two numbers be x and y.

It is given that:

x = 54

HCF = 27

LCM = 162

We know,

x $\times$ y = HCF $\times$ LCM

$\Rightarrow$ 54 $\times$ y = 27 $\times$ 162

$\Rightarrow$ 54y = 4374

$\Rightarrow \quad \therefore y = \frac{4374}{54}$ = 81

2.

(d) 4

Explanation:

f(x) intersects the x -axis at 4 points. hence , f(x) has 4 zeroes.

3. **(a)** (4, 4)

Explanation:

(4, 4)

4.

(c) Real and Equal roots

Explanation:

D = b^2 - 4ac

D = $(-20)^2$ - 4 $\times$ 4 $\times$ 25

D = 400 - 400

D = 0. Hence Real and equal roots.

5.

(b) -504

Explanation:

$S_n = \frac{n}{2} \times [2a + (n - 1)d]$

a = 16, d = -4, and n = 21.

$S_n = \frac{21}{2} \times [2(16) + (21 - 1)(-4)]$

$= \frac{21}{2} \times [32 + 20 \times (-4)]$

$= \frac{21}{2} \times [32 - 80]$

$= \frac{21}{2} \times (-48)$

Now, calculate the product:

= -21 $\times$ 24

= - 504

So, the sum of the first 21 terms of the given A.P. is -504.

6.

(b) $7\sqrt{2}$

Explanation:

The distance between the points A(-1, 5) and B(6, -2) is

$= \sqrt{(6+1)^2 + (-2-5)^2}$

$= \sqrt{7^2 + 7^2}$

$= \sqrt{49 + 49}$

$= \sqrt{98}$

$= \sqrt{7 \times 7 \times 2} = 7\sqrt{2}$

7.

(d) $AP = \frac{1}{2}AB$

Explanation:

AP = $\sqrt{(2-4)^2 + (1-2)^2}$

= $\sqrt{4+1}$ = $\sqrt{5}$ = units

AB = $\sqrt{(8-4)^2 + (4-2)^2}$

= $\sqrt{16+4}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Here AB = 2 $\times$AP

$\therefore$ AP = $\frac{1}{2}$AB

8.

(b) 5 cm.

Explanation:

In triangles APB and CPD,

$\angle$APB = $\angle$CPD [Vertically opposite angles] $\angle$BAP = $\angle$ACD [Alternate angles as $AB||CD$]

$\therefore \Delta$APB $\sim \Delta$CPD [AA similarity]

$\therefore \frac{\text{AB}}{\text{CD}} = \frac{\text{CP}}{\text{AP}}$

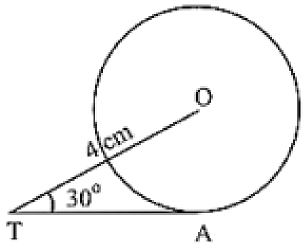
$\Rightarrow \frac{4}{6} = \frac{\text{AP}}{7.5}$

$\Rightarrow$ AP = $\frac{7.5 \times 4}{6}$ = 5 cm

9.

(c) $2\sqrt{3}\ cm$

Explanation:



In right angled triangle OTA,

$\cos 30^\circ = \frac{\text{TA}}{\text{TO}} \Rightarrow$

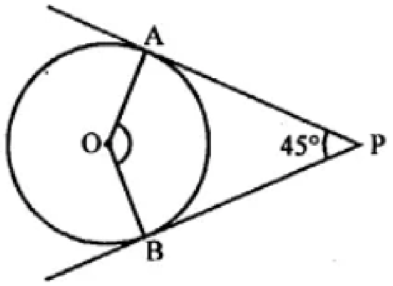
$\frac{\sqrt{3}}{2} = \frac{\text{TA}}{4} \Rightarrow$ TA = $2\sqrt{3}$ cm

10.

(d) 135°

Explanation:

In the given figure, PA and PB are two tangents drawn from an external point P which inclined at an angle of 45°. OA and OB are radii of the circle.



To find $\angle$AOB

$\triangle$OBP is a cyclic quadrilateral

$\angle$AOB + $\angle$APB = 180°

$\Rightarrow \angle$AOB + 45° = 180°

$\Rightarrow \angle$AOB = 180° – 45° = 135°

11. **(a)** $\sqrt{6}$

Explanation:

2 cos45^o cot30^o

$= 2 \times \left(\frac{1}{\sqrt{2}}\right) \times (\sqrt{3})$

$= \frac{2\sqrt{3}}{\sqrt{2}}$

$= \sqrt{6}$

12.

(b) $\frac{3}{4}$

Explanation:

$\sin\theta = \frac{3}{4}$,

$\frac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta} = \frac{tan^2\theta \times cos^2\theta}{sin\theta}$

$= \tan^2\theta \times \cot\theta \times \cos\theta$

$= \tan\theta \times \cos\theta$

$= \frac{sin\theta}{cos\theta} \times \cos\theta$

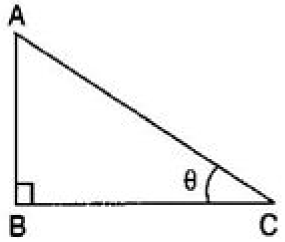
$= \sin\theta$

$= \frac{3}{4}$

13.

(d) 30°

Explanation:



Let Height of the pole AB = x m and length of the shadow BC = $\sqrt{3}x$ meters

$\therefore \tan\theta = \frac{\text{AB}}{\text{BC}}$

$\Rightarrow \tan\theta = \frac{x}{\sqrt{3}x}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^\circ$

$\Rightarrow \theta = 30^\circ$

14.

(d) $\frac{\theta}{720} \times 2\pi r^2$

Explanation:

Area of circle = πr^2

And

Fraction of a sector of angle θ in a circle

$= \frac{\theta}{360^\circ}$

$\therefore$ Area of the sector

$= \frac{\theta}{360} \times \pi r^2$

$= \left(\frac{\theta}{360} \times \pi r^2\right) \times \frac{2}{2} = \frac{\theta}{720} \times 2\pi r^2$

15.

(d) 8 cm

Explanation:

We have given length of the arc and area of the sector bounded by that arc and we are asked to find the radius of the circle.

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$.

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

Now we will substitute the values.

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

$20\pi = \frac{\theta}{360} \times \pi r^2$ ....(1)

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

$5\pi = \frac{\theta}{360} \times 2\pi r$ ....(2)

$\frac{20\pi}{5\pi} = \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r}$

$\frac{20}{5} = \frac{r^2}{2r}$

$\therefore 4 = \frac{r}{2}$

$\therefore$ r = 8

Therefore, radius of the circle is 8 cm.

16.

(d) 1 - p

Explanation:

We know that P(E) + P(not E) = 1

So P(not E) = 1 - P(E)

= 1 - p

17.

(d) $\frac{1}{6}$

Explanation:

Doublet means getting same number on both dice simultaneously

Doublets = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Number of possible outcomes = 6

Total number of ways to throw a dice = 36

Probability of getting a doublet = $\frac{6}{36} = \frac{1}{6}$

18.

(b) 26

Explanation:

mode = 3 median - 2 mean

= 3(30) - 2(32)

= 90 - 64

= 26

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Total Surface Area = CSA of hemisphere + CSA of cone

20.

(d) A is false but R is true.

Explanation:

We have,

$a_n = a + (n - 1)d$

$a_{21} - a_7 = \{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$

a + 20d - a - 6d = 84

14d = 84

$d = \frac{18}{14} = 6$

d = 6

So, A is false but R is true.

Section B

21. Let a and b are numbers and HCF = x

Then LCM = 14x

Now sum of HCF and LCM

x+14x =600

15x = 600

x = 40

Hence HCF=40 and LCM=14×40

Given a=280 and b=?

We know that

$a \times b = \text{HCF} \times \text{LCM}$

So $b = \frac{40 \times 14 \times 40}{280} = 2 \times 40 = 80$

Hence the other number = 80

22. E is the point on side CB produced on an isosceles triangle ABC with AB=AC.AD-BC$\perp$ and EF$\perp$AC. with AB=AC. Also, AD $\perp$ BC and EF$\perp$AC.

To prove: $\triangle$ABD $\sim$ $\triangle$ECF

Proof: In $\triangle$ABD and $\triangle$ECF,

$\therefore$ AB = AC ....Given

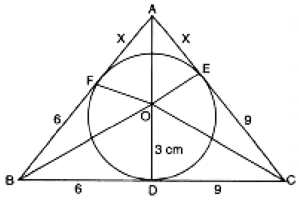
$\therefore$ $\angle$ACB = $\angle$ABC ......Angle opposite to equal sides of a triangle are equal

$\Rightarrow \angle ABC = \angle ACB$

$\Rightarrow \angle ABD = \angle ECF$ ....(1)

$\angle ADB = \angle EFC$ ....(2) [Each equal to 90^0 In view of (1) and (2)]

$\triangle$ABD $\sim$ $\triangle$ECF.............AA similarity criterion

23. 

Let, AF = AE = x

ar $\triangle$ABC = ar$\triangle$AOB + ar$\triangle$BOC + ar$\triangle$AOC

ar $\triangle$ABC = $\frac{1}{2}(15)(3) + \frac{1}{2}(6 + x)(3) + \frac{1}{2}(9 + x)(3)$

$\frac{1}{2}[15 + 6 + x + 9 + x].3 = 54$

45 + 3x - 54

x = 3

$\therefore$ AB = 9 cm, AC = 12 cm

and BC = 15 cm.

24. Dividing N^r & D^r by sin A in LHS

$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$

= cosec A + cot A

OR

Given, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

Taking LCM

$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$

$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$

$\frac{\cos\theta(2)}{\cos^2\theta} = 4$

$\frac{2}{\cos\theta} = 4$

$\cos\theta = \frac{2}{4} = \frac{1}{2}$

$\cos\theta = \cos 60^\circ$

$\therefore \theta = 60^\circ$

25. Radius of circle (r) = OA = 7 cm.

Area of the semicircle = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

= 11 $\times$ 7

= 77 cm^2

Area of $\triangle$ABC = $\frac{1}{2}$ $\times$ base $\times$ height

= $\frac{1}{2} \times 14 \times 7$

= 49 cm^2

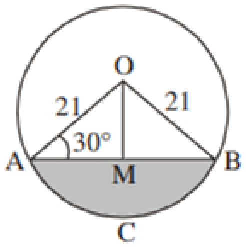
$\therefore$ Area of the shaded portion = Area of semicircle - Area of the $\triangle$ABC

= 77 - 49

= 28 cm^2

OR

Draw OM $\perp$ AB



$\angle$OAB = $\angle$OBA = 30^o

sin 30^o = $\frac{1}{2} = \frac{OM}{21}$ $\Rightarrow$ OM = $\frac{21}{2}$

cos 30^o = $\frac{\sqrt{3}}{2} = \frac{AM}{21}$ $\Rightarrow$ AM = $\frac{21}{2}\sqrt{3}$

Area of $\triangle$OAB = $\frac{1}{2}$ $\times$ AB $\times$ OM = $\frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

= $\frac{441}{4}\sqrt{3}$ cm^2

$\therefore$ Area of shaded region = Area (sector OACB) - Area ($\triangle$OAB)

= $\frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$

= $\left(462 - 441\frac{\sqrt{3}}{4}\right)$ cm^2 or 271.3 cm^2 (approx.)

Section C

26. Let 3 + $5\sqrt{2}$ be rational and have only common factor 1.

Let, 3 + $5\sqrt{2}$ = $\frac{a}{b}$

$5\sqrt{2}$ = $\frac{a}{b}$ - 3

$\sqrt{2}$ = $\frac{a-3b}{5b}$

If $\frac{a-3b}{5b}$ is rational so, $\sqrt{2}$ is also rational number but it is not true as $\sqrt{2}$ is an irrational number.

So it is contradiction to our assumption,

Therefore, $3 + 5\sqrt{2}$ is an irrational number.

27. $\because \alpha$ and β are zeroes of given polynomial

So, $x^2 + 9x + 20 = 0$

$x^2 + 4x + 5x + 20 = 0$

$x(x + 4) + 5(x + 4) = 0$

$(x + 5)(x + 4) = 0$

$x = -5$ and $x = -4$

$\therefore \alpha = -5$ and $\beta = -4$

Now, $\alpha + 1 = -4$ and $\beta + 1 = -3$

So, product of zeroes= $(-4) \times (-3) = 12$

Sum of zeroes = -7

Now polynomial = x^2 - (sum of zeroes)x + (product of zeroes)

Polynomial = $x^2 + 7x + 12$

28. According to the question,

First term of an AP, a=22

Last term=n^{th} term=-11

Sum of n terms = $S_n = \frac{n}{2}(a+l) = 66$

$\Rightarrow \frac{n}{2}(22-11) = 66$ or $\frac{n}{2} \times 11 = 66$

$\therefore n = \frac{66 \times 2}{11} = 12$

n^{th} term = $l = a+(n-1)d$

$\therefore -11 = 22 + (12-1) \times d$ or $-11 = 22+11d$

$\Rightarrow 11d = -22-11$

$\Rightarrow 11d = -33$

$\therefore d = \frac{-33}{11} = -3$

Thus, n=12, d=-3.

OR

Let nth terms of the given arithmetic progressions be t_n and T_n respectively.

The first AP is 13,19,25,....

Let its first term be 'a' and common difference be 'd'. Then,

a = 13 and d = (19 -13) = 6.

We know that in general nth term is given by

$t_n = a + (n-1)d$

$\Rightarrow \quad t_n = 13 + (n-1) \times 6$

$\Rightarrow t_n = 6n + 7$ ....(i)

The second AP is 69,68,67,....

Let its first term be A and common difference be D. Then,

A = 69 and D = (68 - 69) = -1.

Now, we know that in general nth term is given by

$T_n = A + (n - 1) \times D$

$\Rightarrow Tn = 69 + (n - 1) \times (-1)$

$\Rightarrow T_n = 70 - n$ ....(ii)

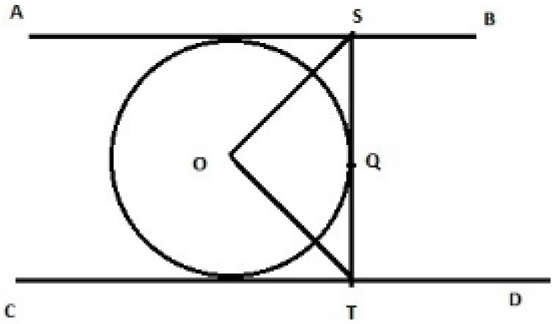
Now, it is given that the two nth terms of the two arithmetic progressions are equal for a value of n, we have,

$t_n = T_n \Rightarrow 6n + 7 = 70 - n$

$\Rightarrow 7n = 63 \Rightarrow n = 9$.

Hence, the 9th term of each AP is the same.

This term = 70 - 9 = 61 [$\because T_n = (70-n)$].

29. 

From the given figure we have, AB⊥ST, then ∠ASQ = 90° and CD⊥TS then ∠CTQ = 90°

$\angle ASO = \angle QSO = \frac{90}{2}$ = 45°

Similarly, $\angle OTQ$ = 45°

To find $\angle SOT$

Consider △SOT

$\angle OTS$ = 45° and $\angle OST$ = 45°

$\angle SOT + \angle OTS + \angle OST$ = 180° (by angle sum property of a triangle)

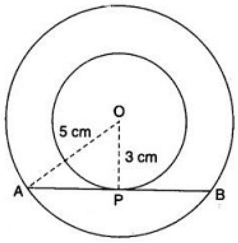
$\angle SOT$ = 180° - ($\angle OTS + \angle OST$) = 180° - (45° + 45°) = 180° - 90° = 90°

Therefore, $\angle SOT$ = 90°

Hence proved

OR

Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA

Then, ∠OPA = 90^o [∵The tangent at any point of a circle is perpendicular to th radius through the point of contact]

∴ $OA^2 = OP^2 + AP^2$ .... By Pythagoras theorem

⇒ $(5)^2 = (3)^2 + AP^2$

⇒ $25 = 9 + AP^2$

⇒ $P^2 = 25 - 9$

⇒ $AP^2 = 16$

⇒ AP = $\sqrt{16}$ = 4 cm

SInce the perpendicular from the centre of a circle to a chord bisects the chord, therfore,

AP = BP = 4 cm

∴ AB = AP + BP = AP + AP = 2AP = 2(4) = 8 cm

Hence, the required length is 8 cm.

30. LHS = $\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$

= sin A cos A

RHS = $\frac{1}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$

$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$

= sin A cos A = LHS

31.

Class interval	Mid value (x)	Frequency (f)	fx	Cumulative frequency
0 – 50	25	2	50	2
50 – 100	75	3	225	5

100 – 150	125	5	625	10
150 – 200	175	6	1050	16
200 – 250	225	5	1127	21
250 – 300	275	3	825	24
300 – 350	325	1	325	25
		N = 25	Σfx = 4225	

Mean = $\frac{\Sigma fx}{N} = \frac{4225}{25}$ = 169

We have,

N = 25

Then, $\frac{N}{2} = \frac{25}{2}$ = 12.5

The cumulative frequency just greater than $\frac{N}{2}$ is 16, then the median class is 150 - 200 such that

l = 150, h = 200 – 150 = 50, f = 6, F = 10

Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h

= 150 + $\frac{12.5-10}{6} \times$ 50

= 150 + $\frac{125}{6}$

= 150 + 20.83

= 170.83

Here the maximum frequency is 6, then the corresponding class 150 - 200 is the modal class

l = 150, h = 200 – 150 = 50, f = 6, f_1 = 5, f_2 = 5

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

= 150 + $\frac{6-5}{2\times6-5-5} \times$ 50

= 150 + $\frac{50}{2}$

= 150 + 25

= 175

Section D

32. Here roots are equal,

$\therefore D = B^2 - 4AC = 0$

Here, $A = 1 + m^2, B = 2mc, C = \left(c^2 - a^2\right)$

$\therefore (2mc)^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$

or, $4m^2c^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$

or, $m^2c^2 - \left(c^2 - a^2 + m^2c^2 - m^2n^2\right) = 0$

or, $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$

or, $-c^2 + a^2 + m^2a^2 = 0$

or, $c^2 = a^2\left(1 + m^2\right)$

Hence Proved.

OR

Let P be the initial production (2 yr ago) and the increase in production in every year be x%.

Then, production at the end of the first year.

$P + \frac{Px}{100} = P(1 + \frac{x}{100})$

Production at the end of the second year = $P(1 + \frac{x}{100}) + \frac{x}{100}P[1 + \frac{x}{100}]$

$= P(1 + \frac{x}{100})(1 + \frac{x}{100})$

$= P(1 + \frac{x}{100})^2$

Since, production doubles in the last two years,

$\therefore P(1 + \frac{x}{100})^2 = 2P$

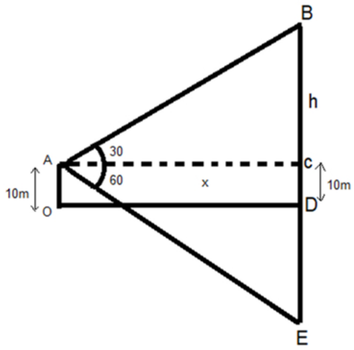
$\Rightarrow (1 + \frac{x}{100})^2 = 2$

$\Rightarrow (1 + \frac{x}{100}) = \sqrt{2}$

$\Rightarrow \frac{x}{100} = \sqrt{2} - 1 = 1.4142 - 1 = 0.4142$

$\Rightarrow x = 0.4142 \times 100$

$\Rightarrow x = 41.42\%$

33. 

Let AC = OD = x, BC = h

$\therefore$ DE = h + 10 (reflection of cloud from surface)

Now, In $\triangle$ABC

tan 30^o = $\frac{BC}{AC}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

x = h$\sqrt{3}$

Also In $\triangle$ACE

tan 60^o = $\frac{CE}{AC}$

$\sqrt{3} = \frac{CD+DE}{AC}$

$\sqrt{3} = \frac{10+h+10}{x}$

$\sqrt{3}$x = 20 + h

$\sqrt{3} \times$ h$\sqrt{3}$ = 20 + h

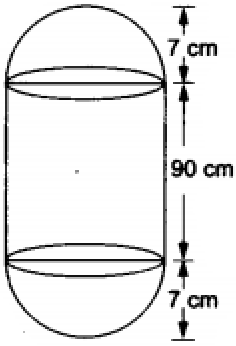
3h = 20 + h

2h = 20

h = 10

Height of cloud from surface of lake = BC + CD

= 10 + 10 = 20m

34. 

Radius of each hemispherical end = 7 cm.

Height of each hemispherical part = its radius = 7 cm.

Height of the cylindrical part = (104 - 2 $\times$ 7) cm = 90 cm.

Area of surface to be polished = 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)

= [2 $(2\pi r^2) + 2\pi rh$] sq units

$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7\right) + \left(2 \times \frac{22}{7} \times 7 \times 90\right)\right] \text{cm}^2$

= (616 + 3960) cm^2 = 4576 cm^2

= $\left(\frac{4576}{10 \times 10}\right)$ dm^2 = 45.76 dm^2 [$\because$ 10 cm = 1 dm].

$\therefore$ cost of polishing the surface of the solid

= ₹(45.76 $\times$ 10) = ₹ 457.60.

OR

Height of cone (h) = 10 cm

Radius of cone and hemisphere (r) = 7 cm

Slant height of cone (l) = $\sqrt{h^2 + r^2}$

l = $\sqrt{10^2 + 7^2}$ = $\sqrt{100 + 49}$ = $\sqrt{149}$

l = 12.2cm

Volume of toy = volume of cone + volume of hemisphere

Volume of toy = $\pi r^2 h + \frac{2}{3}\pi r^3$

Volume = $\pi r^2 \left(h + \frac{2}{3}r\right) = \frac{22}{7} \times 49 \times \left(10 + \frac{2}{3} \times 7\right)$

Volume = $22 \times 7 \times \left(10 + \frac{14}{3}\right)$ = $\frac{22 \times 7 \times 44}{3}$

Volume = 2258.66 cm^3

Volume of toy = 2258.66cm^3

Now,

Surface area of toy = CSA of cone + CSA of hemisphere

Surface area = $\pi rl + 2\pi r^2$

Surface area = $\pi r(l + 2r) = \frac{22}{7} \times 7$ (12.2 + 14)

Surface area= 22 $\times$ 26.2

Surface area = 576.4 cm^2

Surface area of coloured sheet required = 576.4cm^2

35.

Daily expense (CI)	**Households (f_i)**	**Mid-point (x_i)**	**$f_i x_i$**
100-150	4	125	500
150-200	5	175	875
200-250	12	225	2700
250-300	2	275	550
300-350	2	325	650
	$\sum f_i = 25$		$\sum f_i x_i = 5275$

Mean = $\frac{\sum f_i x_i}{\sum f_i}$ = $\frac{5275}{25}$ = 211

For modal expenditure

$\because$ 12 is the maximum frequency

$\therefore$ Modal class = 200 - 250

$\Rightarrow$ l = 200, h = 50, $f_0 = 5$

$f_1 = 12$, $f_2 = 2$

Now,

Mode = l + $\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times$ h

= 200 + $\left(\frac{12-5}{2 \times 12 - 5 - 2}\right) \times$ 50

= 200 + $\left(\frac{7}{24-7}\right) \times$ 50

= 200 + $\left(\frac{7}{17}\right) \times$ 50 = 200 + $\frac{350}{17}$

= 200 + 20.58

Mode = 220.58

Hence, mean daily expenditure = 211

& Modal expenditure - 220.58

Section E

36. i. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

ii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Multiply by 3 in equation (i) and by 4 in equation (ii)

15x + 12y = 28,500 ...(iii)

16x + 12y = 29480 ...(iv)

On subtracting equation (iii) from equation (iv), we get

x = 980

$\therefore$ Prize amount for hockey = ₹ 980

iii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Now, put this value in equation (i), we get

$5 \times 980 + 4y = 9500$

$\Rightarrow 4y = 9500 - 4900 = 4600$

$\Rightarrow y = 1150$

$\therefore$ Prize amount for cricket = ₹ 1150

Difference = 1150 - 980 = 170

$\therefore$ Prize amount for cricket is ₹ 170 more than hockey.

OR

Total prize amount for 2 students each from two games

= 2x + 2y

= 2(x + y)

= 2(980 + 1150)

$= 2 \times 2130$

= ₹ 4260

37. i. Figures A and C are similar.

ii. Only Figure C is congruent.

iii. All congruent figures are similar but all similar figures are not congruent.

For example, a pair of triangles that are similar by the A.A.A. test of similarity are not congruent pairs of triangles since the exact lengths of the sides are unknown.

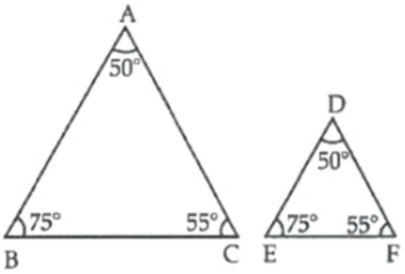
In $\triangle ABC$ and $\triangle DEF$,

$\angle A = \angle D = 50^o$,

$\angle B = \angle E = 75^o$

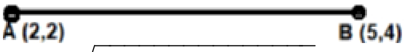
and $\angle C = \angle F = 55^o$.

Hence, $\triangle ABC \sim \triangle DEF$ but they are not congruent.



OR

The length of corresponding sides must be equal.

38. i. 

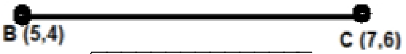
$AB = \sqrt{(5-2)^2 + (4-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

ii. Middle point AB = $\left(\frac{2+5}{2}, \frac{2+4}{2}\right)$

= (3.5, 3)

iii. 

BC = $\sqrt{(7-5)^2+(6-4)^2}$

$=\sqrt{4+4}$

$=2\sqrt{2}$

OR

Middle point of BC = $\left(\frac{5+7}{2}, \frac{4+6}{2}\right)$

= (6, 5)